Contributions of Initial Holdings and Transactions to Performance

While analyzing an investment portfolio and, more specifically, untangling its transactions, you've probably wondered which of these trades have had a decisive influence, positive or negative, on the performance of the portfolio. In this paper, we compute the contribution to performance of each initial holding and each transaction individually, i.e., *these contributions can also be calculated at the level of the individual securities. The result is a time series of contributions to return, revealing transactions or groups of transactions having determinedly influenced the performance of the investment.*

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INTRODUCTION

Traditionally, the excess return is computed relative to the passive strategy; more precisely, relative to an investment that exactly follows the benchmarks of the asset classes and whose weights equal those of the defined investment strategy. This is the case, for example, in the eminent performance attribution in Brinson *et al.* (1986) or the decomposition of the money-weighted rate of return of Illmer and Marty (2003). Cantaluppi (2013) argues that it is equally meaningful to compute the excess return relative to the "do nothing" investment policy. This means that every active decision taken by the asset manager will be considered as performance relevant, independent of the purpose of this decision. In particular, it does not matter whether trades were performed for rebalancing or for selection purposes. The turnover performance defined in Cantaluppi is computed at the level of the asset classes. The arguments of Cantaluppi for basing the excess return on the "do nothing" policy can be summarized as follows:

- The asset manager does not continually rebalance the asset classes even if her expectations are neutral.
- The client has another perception of the active asset management. Each trade is seen as an active decision of the asset manager; whether this was done for rebalancing purposes, or actively over- and underweighting some asset classes, is irrelevant. The main question is, "What has the asset manager done?" and only subsequently, "Why has she done it?"

In this article, we will also consider excess returns relative to the "do nothing" policy, but we will work at the level of individual securities instead of asset classes. To this end, we will compute a value-based performance whose advantages have already been recognized by Armitage and Bagot (2009). Concretely we will show how to compute:

- a contribution value associated with each initial holding
- a contribution value associated to each transaction

such that the value of the portfolio at the end of the performance interval is equal to the value of the portfolio at the beginning of the performance interval, plus all the contributions of the initial holdings and of the transactions during this same performance interval. We refer to this equation as the value equation.

The contributions computed this way are not abstract numbers but relate intuitively to the decisions of the asset manager. They are independent of the investment strategy. They are even independent of the underlying asset classes and their associated benchmarks. Actually, the computation does not even require an investment strategy and benchmarks. Since they are computed at the level of individual securities, they can be aggregated to any desired group, *e.g.*, asset classes. We first compute the contributions of the initial holdings by asking the following question: "What would have been the absolute contribution of any initial holding under the assumption that it would have been kept until the end of the performance interval," similar to a "buy and hold" analysis. We then compute the contribution of each transaction by asking the following question: "What is the difference in the end value of the investment with and without the execution of the transaction?" Of course, we also take into account the contributions of the cash flows. We finally arrive at the "value equation" that gives the end value of the portfolio as the sum of the start value of the portfolio, the contributions of the initial holdings, the contributions of the transactions, and the contributions of the cash flows.

Since the computation of the contributions is quite complex, we start with a simple case and progressively increase the difficulty of the situation. We start with an equity portfolio without cash flows and without corporate actions during the performance interval. We also assume that no interests are paid or received and that no fees are paid. We then proceed to the case of a dividend payment during the performance interval and then extend it to any corporate action. We then show how to generalize the computations to other investment portfolios.

We then show how the value equation relates to the money-weighted rate of return. Since we know the contributions to performance of each initial holding and each transaction, we can compute a time series of excess returns. The resulting graph allows a visualization of the periods with favorable and unfavorable transactions.

A SIMPLE CASE: ONLY PURCHASES AND SALES TRANSACTIONS

First, we show how to compute the contributions of the initial holdings, of the transactions, and of the cash flows in a simple case. We assume we have an equity portfolio with no corporate actions during the performance interval. We also assume that no fees or expenses are paid and no interests are paid or received. In other words there are only purchases and sales transactions during the performance interval.

Let (0, T) be the performance interval, q_t^i the quantity of security i in the portfolio at time t, and p_t^i the price of security i at time t. Let c_t the cash position at time t. The start value of the portfolio, *i.e.*, the value of the portfolio at time 0 is

$$V_0 = \sum_i q_0^i \, p_0^i + c_0$$

Now if we kept this portfolio up to time T without transactions, the value of the portfolio at that time would be

$$V_{0,T} = \sum_i q_0^i \, p_T^i + c_0$$

This is a direct consequence of the assumptions above; in particular that there are no corporate actions during the interval (0, T).

We define the contribution of the initial holding in security i to the performance C_H^i as the difference between the value of the initial holding in this security at time T and at time 0, *i.e.*,

$$C_H^i = q_0^i \, p_T^i - q_0^i \, p_0^i$$

This is nothing but the nominal performance of the initial holding in security i if no transaction is made on this security.

If there were no transactions in (0, T) we would be finished. In this case the holdings in security i at time 0 and at time T would be equal $q_T^i = q_0^i$ and we would also have $c_T = c_0$, and therefore the value equation

$$V_T = V_0 + \sum_i C_H^i$$

would hold; *i.e.*, the value of the portfolio at time T is equal to the value of the portfolio at time 0 plus the sum of the contributions of the initial holdings.

But we do have transactions in the time interval (0, T); in this example, purchase and sale transactions. The principle for the computation of the contribution of a transaction to the nominal performance is the following: compute the value of the holding at time T with the transaction minus that value of the holding without the transaction. Let now see how we handle the contribution to performance of purchase and sale transactions.

First, consider the purchase transaction k occurring at time t_k. A quantity $q_{t_k}^{i_k}$ of security i_k is bought at time t_k for a price of $p_{t_k}^{i_k}$ per piece, *i.e.,* for a total amount of $q_{t_k}^{i_k} p_{t_k}^{i_k}$. If we kept these securities until time T, they would have a value of $q_{t_k}^{i_k} p_T^{i_k}$. We define the contribution to performance of the purchase transaction k C_X^k as the difference between these two values, *i.e.*,

$$q_{t_k}^{i_k} p_T^{i_k} - q_{t_k}^{i_k} p_{t_k}^{i_k}$$

The contribution to performance of the transaction is therefore the nominal performance of the bought securities.

Then consider the sale transaction k occurring at time t_k. A quantity $q_{t_k}^{i_k}$ of security i_k is sold at time t_k for a price of $p_{t_k}^{i_k}$ per piece, *i.e.*, for a total amount of $q_{t_k}^{i_k} p_{t_k}^{i_k}$ If we had kept these securities until time T, they would have had a value of $q_{t_k}^{i_k} p_T^{i_k}$.

We define the contribution to performance of the sale transaction k C_X^k as the difference between these two values, i.e. $q_{t_k}^{i_k} p_{t_k}^{i_k} - q_{t_k}^{i_k} p_T^{i_k}$.

The contribution to performance of the transaction is therefore the nominal performance of the sold securities.

We now show that the value of the portfolio at time T is equal to the value of the portfolio at time 0 plus the sum of the contributions of the initial holdings plus the sum of the contributions of the transactions. To this end, consider the security i and calculate the following sum consisting of the initial value in security i, the contribution of the initial holdings in security i, and the contributions of the transactions on security i:

$$q_0^i p_0^i + \left(q_0^i p_T^i - q_0^i p_0^i\right) + \sum_{k \in P}\left(q_{t_k}^i p_T^i - q_{t_k}^i p_{t_k}^i\right) + \sum_{k \in S}\left(q_{t_k}^i p_{t_k}^i - q_{t_k}^i p_T^i\right)$$

where P is the set of purchase transactions and S is the set of sale transactions.

The term $q_0^i p_0^i$ is the value of the position in security i at time 0. The term

$$\left(q_0^i p_T^i - q_0^i p_0^i\right)$$

is the contribution to the nominal performance of the initial holdings in security i. The term

$$\sum_{k \in P}\left(q_{t_k}^i p_T^i - q_{t_k}^i p_{t_k}^i\right)$$

is the contribution to the nominal performance of the purchase transactions on security i, and the term

$$\sum_{k \in S}\left(q_{t_k}^i p_{t_k}^i - q_{t_k}^i p_T^i\right)$$

is the contribution to the nominal performance of the sale transactions on security i. Rearranging the sum above we get

$$q_0^i p_T^i + \sum_{k \in P}\left(q_{t_k}^i p_T^i\right) - \sum_{k \in S}\left(q_{t_k}^i p_T^i\right) - \sum_{k \in P}\left(q_{t_k}^i p_{t_k}^i\right) + \sum_{k \in S}\left(q_{t_k}^i p_{t_k}^i\right)$$

Now the sum of the first three terms equals $q_T^i p_T^i$, since q_T^i (the quantity held in security i at time T) is equal to the quantity of this security held at time 0 plus the quantities bought minus the quantities sold during the interval (0, T). The term $q_T^i p_T^i$ equals the value of the position in security i at time T. In order to get the sum of initial values and contributions for the whole investment, we have to sum this equation for all securities i and add c_0, the cash position at time 0. We therefore get

$$\sum_i q_T^i p_T^i + c_0 + \sum_i\left(\sum_{k \in S}\left(q_{t_k}^i p_{t_k}^i\right) - \sum_{k \in P}\left(q_{t_k}^i p_{t_k}^i\right)\right)$$

The last sum are the cash movements resulting from the purchase and sale transactions on all securities. Since there are only purchase and sale transactions, without fees or interests paid or received we have

$$c_T = c_0 + \sum_i\left(\sum_{k \in S}\left(q_{t_k}^i p_{t_k}^i\right) - \sum_{k \in P}\left(q_{t_k}^i p_{t_k}^i\right)\right)$$

We finally have that the sum of the value of the portfolio at time 0 plus the sum of the contributions of the initial holdings plus the sum of the contributions of the transactions equals to

$$\sum_i q_T^i p_T^i + c_T$$

which is the value of the investment at time T.

We therefore have

$$V_T = V_0 + \sum_i C_H^i + \sum_k C_X^k$$

showing that the value of the portfolio at time T is equal to the value of the portfolio at time 0 plus the contributions of the initial holdings plus the contributions of the transactions.

If there were cash flows during the period (0, T), we would define the contribution C_F^j of the cash flow j simply as the value of the cash flow. It is then easy to see that the value equation

$$V_T = V_0 + \sum_i C_H^i + \sum_k C_X^k + \sum_j C_F^j$$

holds.

EXAMPLE WITH PURCHASES AND SALES ONLY

The performance interval is the first half of the year 2013. The positions of the portfolio at the end of 2012 are shown in Table 1.

The transactions during the period are:

- 10 pieces of "Security 2" are sold at a price of $42 each on March 3, 2013.
- 10 pieces of "Security 1" are bought at a price of $48 each on April 14, 2013.

The prices of the securities on June 30, 2013 are given below:

Security 1: $52
Security 2: $38
Security 3: $35

The contributions of the initial holdings to the value performance are shown in Table 2.

The contributions of the two transactions to the value performance are shown in Table 3.

The cash balance on June 30, 2013 is $40 = 100 (start balance) + 420 (sale of 03.03.2013) - 480 (purchase of

Table 1

12.31.2012	Quantity	Price	Value
Security 1	10	50	500
Security 2	20	40	800
Security 3	20	30	600
Cash	100	1	100
Total value			2'000

Table 2

	Quantity	Price 12.31.2012	Price 06.30.2013	Contribution
Security 1	10	50	52	10 * (52 - 50) = 20
Security 2	20	40	38	20 * (38 - 40) = -40
Security 3	20	30	35	20 * (35 - 30) = 100
Total value				80

Table 3				
	Quantity	Price trx	Price 06.30.2013	Contribution
Sale of Security 2 03.03.2013	10	42	38	10 * (42 - 38) = 40
Purchase of Security 1 04.14.2013	10	48	52	10 * (52 - 48) = 40
Total value				80

04.14.2013). Therefore, the positions of the portfolio on June 30, 2013 are as follows (see Table 4).

We can verify that the value performance of \$160 = 2,160 - 2,000 is equal to the sum of the contributions of \$160 = 80 (contributions of initial holdings) + 80 (contributions of transactions). Considering the initial investment of \$2'000 we have a total performance of 8% (= 160 / 2'000) that we can breakup into the following contributions (see Table 5).

WITH A DIVIDEND PAYMENT

We now assume that a security i, present in the portfolio in at least part of the time interval (0, T), has a dividend payment of d^i per piece at time t. Of course the dividend transaction contributes to the performance, but the collecting of the dividend is a consequence of previous holdings and transactions. The contribution of the dividend to performance, therefore, must be attributed to the holdings and transactions that have resulted in the holdings giving the dividend payment.

We assume that we have computed the contributions of initial holdings and the contributions of transactions as in the simple case presented above. We then update these contributions according to the rules below. The sum of these updates corresponds to the exact dividend payment. The cash received as dividend payment is computed as contribution of holdings and transaction to the nominal performance.

- Update the contribution of the initial *i* holdings in security i by adding a value of d^i per piece.
- Update the contribution of all purchases of security i before the dividend date by adding a value of d^i per piece purchased.
- Update the sales of security i before the dividend date by subtracting a value of d^i per piece sold.

Notice that the update of the contributions has to be done even if the whole holding in security i has been sold before the dividend date. Although the total nominal performance would otherwise also be correct, the contributions of the transactions would not have the desired intuitive explanation. The contributions of hold-

Table 4			
06.30.2013	Quantity	Price	Value
Security 1	20	52	1'040
Security 2	10	38	380
Security 3	20	35	700
Cash	40	1	40
Total value			2'160

Table 5

Event	Contribution in $	Contribution in %
Initial holding in Security 1	20	1%
Initial holding in Security 2	-40	-2%
Initial holding in Security 3	100	5%
Total initial holdings	80	4%
Sale of Security 2 of 03.03	40	2%
Purchase of Security 1 of 04.14	40	2%
Total value	160	8%

Table 6

	Contribution without dividend	Contribution of dividend	Total contribution
Security 1	10 * (52 - 50) = 20	0	20
Security 2	20 * (38 - 40) = -40	20 * 2 = 40	0
Security 3	20 * (35 - 30) = 100	0	100
Total value	80	40	120

ings and purchases in security i would be undervalued and the contribution of sales in security i would be overvalued, thus distorting the valuation of the decisions of the asset manager. The same principle can be used for the cash component of any corporate action.

EXAMPLE WITH A DIVIDEND

Let us go back to the simple example above with the following extension: a dividend payment on "Security 2" of $2 per piece on May 8, 2013. The holdings for "Security 2" at this date are 10 pieces; we therefore receive a total of $20 in dividend payment. The dividend payment is a consequence of the holdings at the date of the dividend; *i.e.*, the consequence of previous trades. Thus the contribution of the dividend to the performance has to be added to the contributions of previous holdings and trades. In this example, the holdings of 20 pieces of "Security 2" at the end of 2012 would have resulted in a dividend payment of $40 on May 8, 2013. The contribution to the nominal performance of this initial holding must be increased by a value of $40, making the total contribution of this initial holding equal to 0. The sale of 10 pieces of "Security 2" on March 3, 2013 resulted in a decrease of the dividend payment of $20. The contribution to the nominal performance of this transaction must, therefore, be reduced by that amount, making this total contribution equal to 20.

The contributions of the initial holdings are summarized in Table 6.

The contributions of the transactions are summarized in Table 7.

The holdings on June 30, 2013 are given in Table 8. Compared to the example above, only the cash balance has changed, increasing by $20, reflecting the dividend payment.

Here, again, we can verify that the sum of the contributions of $180 equals the increase in value of the investment portfolio. With an initial investment of $2,000 this performance is 9% (= 180 / 2,000) whose breakup is given in Table 9.

Compared to the simple example above, we see that the contribution of Security 2 to performance has increased, reflecting the received dividend. The initial holding in Security 2 has a contribution increased by 2% and the

Table 7

	Contribution without dividend	Contribution of dividend	Total contribution
Sale of Security 2 03.03.2013	10 * (42 - 38) = 40	- 10 * 2 = -20	20
Purchase of Security 1 04.14.2013	10 * (52 - 48) = 40	0	40
Total value	80	-20	60

Table 8

06.30.2013	Quantity	Price	Value
Security 1	20	52	1'040
Security 2	10	38	380
Security 3	20	35	700
Cash	60	1	60
Total value			2'180

Table 9

Event	Contribution in $	Contribution in %
Initial holding in Security 1	20	1%
Initial holding in Security 2	0	0%
Initial holding in Security 3	100	5%
Total initial holdings	120	6%
Sale of Security 2 of 03.03	20	1%
Purchase of Security 1 of 04.14	40	2%
Total value	180	9%

sale of Security 2 before collecting the dividend decreased its contribution by 1 percent.

WITH ANY CORPORATE ACTION

The principle for the calculation of the contributions is the same as in the simpler cases described above. This means that the computation of the contributions concerning security i necessitates the price of security i at time T. If there has been a corporate action for security i during the interval (0, T), the price of security i at time T is either not known because security i does not exist anymore (*e.g.*, after a merge), or the price of security i does not reflect the reality because security i at time T does not represent the same entity (*e.g.*, after a split or a spin off). We will, therefore, replace the price of security i at time T by the weighted prices at time T of the securities arising from the corporate actions, the weights being given by the corporate action. If the corporate action includes a cash component, it will be handled the same way the dividend payment is handled in the case described above; *i.e.,* it increases the contributions of the initial holdings and purchases prior to the corporate action date and decreases the contributions of sales prior to the corporate action date.

Let us formalize this process. Assume that we have a complex corporate action for security i at time t. One unit of security i gives c units of cash and q_j units of security j for a given set of securities j that might include security i. The price p_T^i of security i at time T is replaced by the expression

$$\sum_j q_j \, p_T^j$$

Of course one of the securities j could itself have a corporate action between the time of the corporate action of i and time T. This problem can be easily solved by working the corporate actions backward from time T to time 0. As mentioned above, the cash part of the corporate action is handled separately, similarly to a dividend payment.

Now that we have "prices" at time T for all securities in the portfolio at time T, we can compute the contributions of initial holdings and transactions to the nominal performance as we did for the simpler cases.

EXAMPLE WITH CORPORATE ACTIONS

The performance interval is the first half of the year 2013. The positions of the portfolio at the end of 2012 are shown in Table 10.

The transactions during the period are:

- 20 pieces of "Security 2" are sold at a price of $48 each on February 3, 2013.
- 20 pieces of "Security 1" are bought at a price of $36 each on March 2, 2013.

The following corporate actions occur during the period:

- On 20th March 2013, a spin off operation gives one piece of the new "Security 3" for each holding of 10 pieces of "Security 2."
- On 14th April 2013, "Security 2" under goes a split, with a ratio of 2-for-1.

The prices of the securities on June 30, 2013 are given below:

Security 1: $32
Security 2: $24
Security 3: $40

One piece of "Security 2" on 31st December 2012 will give 0.1 piece of "Security 3" (due to the spin off) and two pieces of "Security 2" (due to the split) on 30th June 2013, for a total value of $52 = 0.1 x 40 + 2 x 24. We can use this value as the "price" of "Security 2" on 30th June 2013 for the initial holding and the sale of 3rd February 2013.

The contributions of the initial holdings to the value performance are shown in Table 11.

The contributions of the two transactions to the value performance are as follows (see Table 12).

The cash balance on June 30, 2013 is $340 = 100 (start balance) + 960 (sale of 02.03.2013) - 720 (purchase of 03.02.2013). Therefore, the positions of the portfolio on June 30, 2013 are as follows in Table 13.

Here, again, we can verify that the sum of the contributions of -$160 equals the decrease in value of the invest-

Table 10

12.31.2012	Quantity	Price	Value
Security 1	40	35	1'400
Security 2	60	50	3'000
Cash	100	1	100
Total value			4'500

Table 11

	Quantity	Price 12.31.2012	Price 06.30.2013	Contribution
Security 1	40	35	32	40 * (32 - 35) = -120
Security 2	60	50	52	60 * (52 - 50) = 120
Total value				0

Table 12

	Quantity	Price trx	Price 06.30.2013	Contribution
Sale of Security 2 02.03.2013	20	48	52	20 * (48 - 52) = -80
Purchase of Security 1 03.02.2013	20	36	32	20 * (32 - 36) = -80
Total value				-160

Table 13

06.30.2013	Quantity	Price	Value
Security 1	60	32	1'920
Security 2	80	24	1'920
Security 3	4	40	160
Cash	340	1	340
Total value			4'340

ment portfolio. With an initial investment of $4,500 this performance is -3.56% (= -160 / 4,500) whose breakup is given in Table 14.

POSSIBLE EXTENSIONS

We have seen how to compute the contributions to the nominal performance for equities. This can be easily extended to other types of investments. Bonds, for example, can be handled in a similar way, treating the interest payments like the dividend payments of equities and setting the price of the bond at time T equal to its redemption price if it matures before time T. This similarity extends to other types of investments like time deposits, mutual funds, etc.

The computations we have presented so far ignored all transaction fees and other expenses. These fees and expenses can easily be taken into account. The transaction fees decrease the contribution to return of the corresponding transaction, and the global fees are considered as negative contributions to return themselves. In order to get the clearest results, it would be desirable to include a separate category of contribution: fees and expenses. All nominal fees and expenses would then be added to this category during the computation process and subsequently be transformed into percent figures. The performance before and after fees would then be directly available.

Of course a real world implementation of this method would require the detailed description of the computation for all types of investments in the portfolios that are processed.

CONTRIBUTION TIME SERIES

We would like to transform the computed nominal performance in a performance expressed in percent, as is usually the case. We first rewrite the value equation

$$V_T = V_0 + \sum_i C_H^i + \sum_k C_X^k + \sum_j C_F^j$$

as

$$V_T - V_0 - \sum_j C_F^j = \sum_i C_H^i + \sum_k C_X^k$$

Notice then that the left part of the second equation is the numerator in the Modified Dietz formula, which computes the return of the portfolio as

$$R_{Mod\ Dietz} = \frac{V_T - V_0 - \sum_j C_F^j}{V_0 + \sum_j w_F^j C_F^j}$$

where

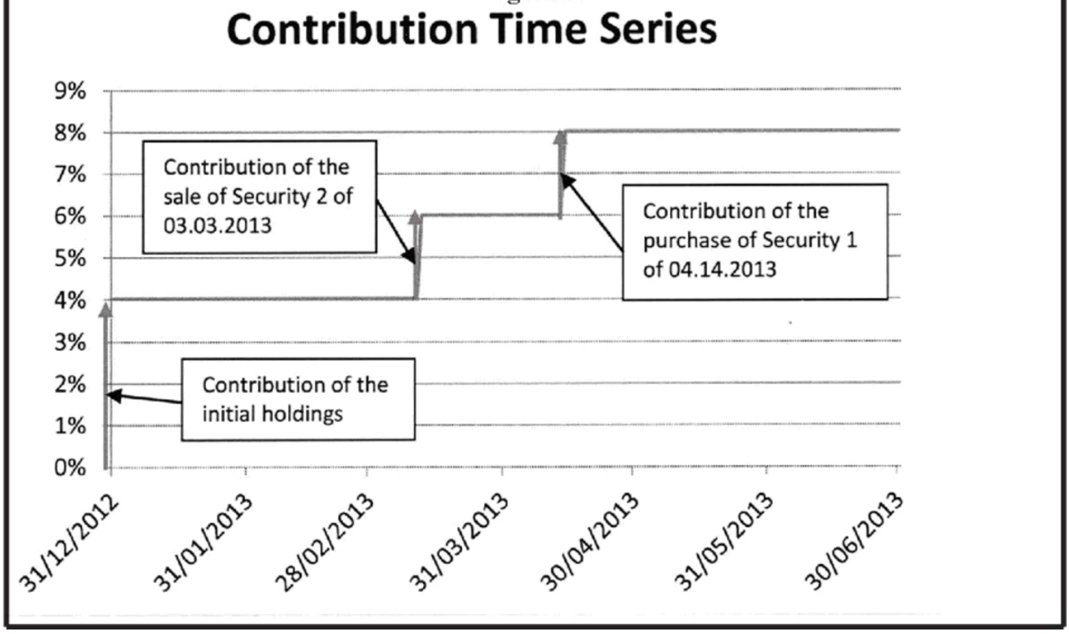
$$w_F^j = \frac{T - t^j}{T}$$

if t^j is the time at which the cash flow j occurs. The denominator of the Modified Dietz formula is referred to as the average (invested) capital. We can, therefore, divide the contributions of the initial holdings and the contributions of the transactions by the average capital to obtain a performance in percent. By doing so we not only know the money-weighted rate of return of the portfolio as computed by the Modified Dietz formula, we also know where it comes from since we know the contribution of each initial holding and each transaction to this result. We can, therefore, cumulate the contributions up to any time t and present the final result not simply as a return. but as a time series of contributions to return. The value of the time series at time 0 is the contribution to performance of the initial holdings, and the value at time T is the total performance for the given period. The change of value of the time series at a given date is the sum of the contributions to return of all trans-

Table 14

Event	Contribution in $	Contribution in %
Initial holding in Security 1	-120	-2.67%
Initial holding in Security 2	120	2.67%
Total initial holdings	0	0%
Sale of Security 2 of 02.03	-80	-1.78%
Purchase of Security 1 of 03.02	-80	-1.78%
Total value	-160	-3.56%

Figure 1



actions at that date.

The resulting graph allows a visualization of the periods with favorable and unfavorable transactions. The sharp changes of the curve readily underline the important transactions of groups of transactions which can then be analyzed in detail. The simple example above (with purchase and sale only) gives the following graph (Figure 1).

The contribution time series starts with a value of 4%, reflecting the contribution of the initial holdings to performance and ends up with a value of 8%, the total performance on this interval. The two increases on March 3 and April 14 are the contributions to performance of the corresponding transactions.

CONCLUSION

The method described in this article breaks up the money-weighted return down to its smallest components: the initial holdings and the transactions. We agree with Davis and Spaulding (2011) who stated, "the best choice for client reporting is actually a money-weighted return." Now if we can on top of that show the client how this money-weighted return has been attained, we have brought the client reporting to a new level of transparency.

REFERENCES

Armitage, Seth, and Gordon Bagot, "Value-based Performance Measurement: A Further Explanation," *The Journal of Performance Measurement*, Winter, pp. 58-74, 2009.

Brinson, Gary, Randolph Hood, and Gerard Beebower, "Determinants of Portfolio Performance," *Financial Analyst Journal*, July - August, pp. 39-44, 1986.

Cantaluppi, Laurent, "Turnover Performance," *The Journal of Performance Measurement*, Spring, pp. 67-74, 2013.

Davis, Trevor, and David Spaulding, "The Myth of GIPS-money-weighted Returns for Client Performance Reporting," *The Journal of Performance Measurement*, Fall, pp. 33-36, 2011.

Illmer, Stefan, and Wolfgang Marty, "Decomposing the Money-Weighted Rate of Return," *The Journal of Performance Measurement*, Summer, pp. 42-50, 2003.